Mathematical & Computational Sciences
UNIVERSITY OF TORONTO
MISSISSAUGA

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Letter of Recommendation for
William Verreault

This letter is provided by the Department of Mathematical and Computational Sciences, University of Toronto Mississauga, to strongly support William Verreault's application to participate in the Hong Kong Laureate Forum. William is an exceptional young Analyst with remarkable mathematical maturity and talent.

William has accomplished a great deal in Analysis and Operator Theory. He has published ten papers covering a variety of applications, from signal processing to pure mathematics in Number Theory. This is an impressive record, especially for someone who has only just begun his Ph.D. studies.

The following lists some of his achievements.

Recent applications of probability in number theory have led to remarkable results. Many multiplicative functions are believed to exhibit random behavior, with their partial sums closely associated with the behavior of L-functions, including the Riemann zeta function. Mathematicians investigate these phenomena using models known as random multiplicative functions (RMFs), which provide insight into the relationship between randomness and the multiplicative structure of integers. Surprisingly, the Riemann hypothesis is expected to hold almost surely for RMFs modeled as a coin flip, although this approach must consider the multiplicative dependency of prime numbers, adding layers of complexity. Notably, the sums of RMFs are known to deviate from a normal distribution, and their moments have been only partially determined.

Recent breakthroughs in this field have emerged from connections with random matrix theory and studies of log-correlated Gaussian fields, such as the Gaussian free field (GFF). In the critical case, it is anticipated that the total mass of multiplicative chaos derived from the GFF reflects the behavior of sums of RMFs. William's research drew on these connections to compute the natural moments of RMFs and to analyze the distribution of subsets of {1, 2, ..., x} that display Gaussian behavior, employing both complex analytic and probabilistic techniques. He has also proposed a conjecture concerning the first moment of these partial sums, which is tied to the open problem of determining their distribution, a question that recent work on multiplicative chaos further supports and brings closer to resolution.

Another area of William's interest was the study of nonlinear expansions in function spaces. In recent years, many mathematicians have focused on a nonlinear analogue of Fourier series that enables the

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approximation of a signal as a sum of terms, with each component representing frequency and amplitude. This is exemplified by the Blaschke unwinding series introduced by Coifman, also known as adaptive Fourier decomposition. Due to its numerous advantages over classical Fourier series, this expansion has found applications in various other problems. However, the question of the series' convergence has remained a significant challenge for several decades.

William demonstrated that the series converges in any reproducing kernel Hilbert space and in crucial Hardy spaces, encompassing not only all previously known cases but also a range of other analytic function spaces that were previously challenging to access. His proposed expansion is more general than the Blaschke unwinding series, as evidenced by comparisons to the de Branges-Rovnyak spaces. Notably, he employs the robust theory of operators and function spaces, which enables simple constructions and proofs of these results. His method provides strong justification for earlier approaches and additionally offers approximation algorithms that classical methods did not present. This breakthrough has opened new avenues for implementing these approximation schemes and further exploring discoveries within the field.

Currently, William is working in the field of Stochastic (or Schramm) Loewner Evolution (SLE) and its relation to various critical planar lattice models of Statistical Physics. SLE, introduced by Oded Schramm in 1998, gained immense popularity over the last two decades in Mathematics and Theoretical Physics. These curves are conjectured to be the scaling limits of the various interfaces in critical lattice models. While some of these conjectures have been verified in recent years, most of them are still open and subject to intense research. The leaders of the field, Wendelin Werner (2006), Stanislav Smirnov (2010), and Hugo Duminil-Copin (2022), have been awarded Fields medals, which is a testament to the importance of SLE in the broad mathematical community. The circle of problems became increasingly popular in the last two decades.

Specifically, William works on understanding Schramm Loewner Evolution from the point of view of the GFF and its various smooth approximations. His research is shedding new light on our knowledge of Liouville Quantum Gravity. He is using his strong Complex Analytic background to advance our understanding of deep Physics phenomena.

William has an outstanding record of service to the community. His contributions to the Canadian Mathematical Society (CMS) are truly remarkable. He has served as the chair of the Student Committee and continues to be a member of that committee. Additionally, he is a member of the Education Committee. For two years, he held the role of Student Director on the Executive Committee. Throughout all his positions, he has demonstrated exceptional leadership skills and unwavering dedication. The list of workshops and conferences William organized is absolutely astonishing. He organized nine (!)

sections and mini-courses at the CMS semi-annual meetings. William was the driving force behind the creation of the Ontario Graduate Mathematics Conference. The conference attracted over 150 students at its inaugural run this year. I am also impressed with another of William's projects, the Directed Reading Program. The program pairs undergraduate and graduate students with similar interests for weekly individual meetings. It is hard to overestimate the importance of this program in retaining talented undergraduate students in mathematics, especially students from underrepresented groups.

In summary, William is an outstanding young researcher. The field of SLE has attracted several bright young researchers who are not afraid to work on difficult interdisciplinary problems. Given his mathematical maturity, curiosity, and ability, we are confident that William will be able to join their ranks. The Department of Mathematical and Computational Sciences recommends William for the Hong Kong Laureate Forum.

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